Incomplete Block Designs
Growth Chamber Example

Objective:
Determine the effect of soil temperature on emergence of *Miscanthus* shoots.

Treatments:
15 °C
20
25
30

Only two growth chambers were available so it was necessary to use an incomplete block design.

Boersma and Heaton, GCB Bioenergy 4:680-687.

Incomplete Block Designs
Growth Chamber Example

Chamber	1	2		1	2
Period 1	25	30	Period 2	15	20
Period 3	15	25	Period 4	20	25
Period 5	20	30	Period 6	15	30

Only two temperature treatments could be tested during a growth period. Treatments were applied to chambers so that every temperature occurred once with every other temperature in the same period so t(t-1)/2 = 6 periods were required. Each treatment was ∴ replicated (t-1) = 3 times.

Incomplete Block Designs

Growth Chamber Example

Source	df	SS
Total	n – 1	$\sum(y_{ij} - \bar{y}_{..})^2$
Incomplete Blocks	b – 1	$k\sum(\bar{y}_{.j} - \bar{y}_{..})^2$
Treatments (adjusted)	t - 1	$\frac{k\sum Q_i^2}{\lambda t}$
Error	n - t - b + 1	TotSS - BSS -TrtSS

$Q_i = y_{i.} - (B_i / k)$

B_i = sum of block totals that include the treatment

k = # experimental units/block   λ = associate class

Incomplete Block Designs

Growth Chamber Example

Source	df	SS	MS	F
Total	11	10767		
Period	5	3767	753.33	
Temperature (adjusted)	3	5900	1966.67	5.36
Error	3	1100	366.67	

Incomplete Block Designs
Growth Chamber Example

SAS Code:

```
proc mixed;
  class Per Temp;
  model Emerg = Per Temp;
  lsmeans Temp;
  contrast 'Temp linear' Temp -3 -1 1 3;
  contrast 'Temp quad' Temp 1 -1 -1 1;
run;
```

Incomplete Block Designs
Growth Chamber Example

SAS Output:

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
Per	5	3	1.44	0.4069
Temp	3	3	5.36	0.1006

Contrasts				
Label	Num DF	Den DF	F Value	Pr > F
Temp linear	1	3	15.71	0.0287
Temp quad	1	3	0.14	0.7364

Incomplete Block Designs

Growth Chamber Example

Adjusting Means for Block Effects

$$\bar{Y}_i = \bar{Y}_{..} + \frac{kQ_i}{\lambda t} = 51.67 + \frac{2Q_i}{4}$$

Treatment	Treatment Means	Q	Adjusted Means
15	13.33	-80	11.67
20	53.33	-10	46. 67
25	60	20	61. 67
30	80	70	86. 67

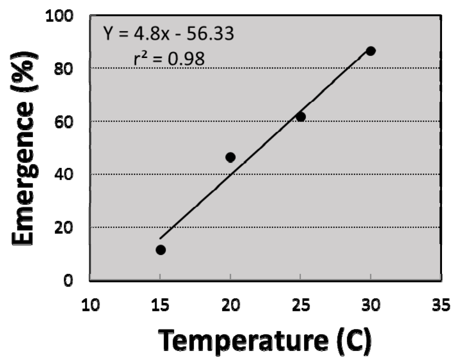
Incomplete Block Designs

Growth Chamber Example

Standard Error of a Difference

$$S_{\bar{d}} = \sqrt{\frac{2kMSE}{\lambda t}} = \sqrt{\frac{2(2)366.67}{1(4)}} = 19.15$$

Incomplete Block Designs
Growth Chamber Example



Lattice Designs
Balanced 3 x 3 x 4 Lattice

Block \ Rep	1			2			3			4		
1	1	2	3	1	4	7	1	5	9	1	8	6
2	4	5	6	2	5	8	7	2	6	4	2	9
3	7	8	9	3	6	9	4	8	3	7	5	3

k = number of treatments / incomplete block

s = k = number of incomplete blocks / rep

t = sk = k^2 = number of treatments / rep

r = k + 1

Lattice Designs
Balanced Sizes

t	9	16	25	49	64	81
k	3	4	5	7	8	9
r	4	5	6	8	9	10

s = number of incomplete blocks / rep
k = number of treatments / block

Lattice Designs
Linear Additive Model

$$Y_{ijk} = \mu + R_i + B_{(i)j} + T_k + \varepsilon_{ijk}$$

Where:

Y_{ijk} = variable to be analyzed from i^{th} rep and j^{th} block
μ = overall mean
R_i = effect of the i^{th} rep
$B_{(i)j}$ = effect of the j^{th} block within the i^{th} rep
T_k = effect of the k^{th} treatment
ε_{ijk} = residual error of ijk^{th} observation

Lattice Designs
Expected Mean Squares

Source	EMS
R_i	$\sigma^2_\varepsilon + k\sigma^2_B + t\sigma^2_R$
$B_{(i)j}$	$\sigma^2_\varepsilon + kr\sigma^2_B$
T_k (unadj.)	$\sigma^2_\varepsilon + [k/k+1]\sigma^2_B + r\Phi T$
T_K (adj.)	$\sigma^2_\varepsilon + r\Phi T$
ε_{ijk}	σ^2_ε

Lattice Designs
ANOVA

Source	df	SS
Replicates	r – 1	$sk\sum(\overline{y}_{i..} - \overline{y}_{...})^2$
Blocks / reps	r(s - 1)	$k\sum(\overline{y}_{ij.} - \overline{y}_{...})^2$
Treatments (adjusted for blocks)	t - 1	$\frac{k\sum Q_i^2}{\lambda t}$
Error	N - t - rs + 1	TotSS - RSS - BSS -TSS

$Q_i = y_{..k} - (B_k / k)$

B_k = sum of block totals that include the k^{th} treatment

Lattice Designs

SAS Analysis

```
proc mixed;
  class rep blk trt;
  model yld = trt;
  random rep blk(rep);
  lsmeans trt / pdiff;
run;
```

Lattice Designs

Partially Balanced

Types:

Simple Lattice – 2 reps
Triple Lattice – 3 reps
Quadruple Lattice – 4 reps

Lattice Designs
Partially Balanced

Constraints:

k = number of treatments / incomplete block

s = k = number of incomplete blocks / rep

t = sk = k^2 = number of treatments / rep

r = 2, 3, or 4 = number of replicates

Lattice Designs
5 x 5 Simple Lattice

Block	Rep 1					Rep 2				
1	1	2	3	4	5	1	6	11	16	21
2	6	7	8	9	10	2	7	12	17	22
3	11	12	13	14	15	3	8	13	18	23
4	16	17	18	19	20	4	9	14	19	24
5	21	22	23	24	25	5	10	15	20	25

Lattice Designs
Partially Balanced

Associate Classes:

$\lambda = 0$ for treatments that never occur together in an incomplete block

$\lambda = 1$ for treatments that occur together once in an incomplete block

Lattice Designs
Partially Balanced

Size	# Treatments										
Simple	9	16	25	36	49	64	81	100	121	144	169
Triple	9	16	25	36	49	64	81	100	121	144	169
Quadruple	9	16	25	36	49	64	81	100	121	144	169

$t = sk = k^2$ = number of treatments / rep
k = number of treatments / incomplete block
s = k = number of incomplete blocks / rep

Lattice Designs

Rectangular Lattice

<u>Constraints</u>:

k = s - 1 = number of treatments / incomplete block

s = k + 1 = number of incomplete blocks / rep

t = sk = number of treatments / rep

r = 2, or 3 = number of replicates

Simple rectangular lattice r = 2

Triple rectangular lattice r = 3

Lattice Designs

3 x 4 x 3 Rectangular Lattice

Block	Rep 1			Rep 2			Rep 3		
1	1	2	3	1	8	11	1	5	7
2	4	5	6	2	5	12	2	9	10
3	7	8	9	3	6	9	3	4	11
4	10	11	12	4	7	10	6	8	12

Lattice Designs
Rectangle Sizes

t	12	20	30	42	56	72	90
s	4	5	6	7	8	9	10
k	3	4	5	6	7	8	9

s = number of incomplete blocks / rep
k = number of treatments / block

Lattice Designs
Available Treatment Sizes

t	9	12	16	20	25	30	36	42	49	56	64	72	81	90	100	144
s	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	12
k	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	12

s = number of incomplete blocks / rep
k = number of treatments / block

Generalized Lattice Designs
Alpha Lattice Designs

Constraints:

k = number of treatments / incomplete block

s = number of incomplete blocks / rep

t = sk = number of treatments / rep

r = 2, 3, or 4 = number of replicates

Generalized Lattice Designs
Some Potential Alpha Designs

t	**15**	**18**	**21**	**24**	**28**	**32**	**40**	**50**	**88**	**120**	**130**	**165**
s	5	6	7	6	7	8	8	10	11	12	13	15
k	3	3	3	4	4	4	5	5	8	10	10	11

s = number of incomplete blocks / rep
k = number of treatments / block

Lattice Designs

Homework Problem

```
proc mixed data=a method=type3;
  class group block treatmnt;
  model yield = treatmnt;
  random group block(group);
  estimate '1 vs. 2' treatmnt 1 -1;
  estimate '1 vs. 7' treatmnt 1 0 0 0 0 0 -1;
  lsmeans treatmnt;
run;
```

Type 3 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Treatmnt	24	711.120000	29.630000	1.97	0.0824
Group	1	212.180000	212.180000	1.90	
Block(Group)	8	501.840000	62.730000	4.59	
Residual	16	218.480000	13.655000		

Lattice Designs

Homework Problem

Covariance Parameter Estimates

Cov Parm	Estimate
Group	4.0150
Block(Group)	19.6300
Residual	13.6550

Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|
1 vs. 2	2.0952	3.9739	16	0.53	0.6052
1 vs. 7	9.9933	4.2342	16	2.36	0.0313